SECTION V: ADDENDUM

PRELIMINARY ANALYSIS OF TEMPERATURE DATA ON STAINED GLASS WINDOWS WITH PROTECTIVE GLAZING

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Preliminary Analysis of Temperature Data on Stained Glass Windows With Protective Glazing

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Introduction

The National Park Service Protective Glazing Study conducted by Inspired Partnerships has provided a unique opportunity for the quantitative analysis of the effect of protective glazing. Dataloggers were installed in windows with protective glazing at St. John U. C. C. in Evanston. Only a small portion of the available data has been analyzed. Preliminary analysis has concentrated on data from two similar windows with the same geometry, both with protective glazing, but one with venting. This gives a direct measure of the effect of venting.

Analysis

The first requirement of this analysis was the calculation of the solar input (insolation) for any location on the earth, any date, any time, and any orientation (elevation and azimuth) of the surface receiving the solar input. Exact calculations would require orbital mechanics, but a good approximation was found which was programmed in the HP keystroke language and had an associated writeup of the equations. These equations were then programmed in Visual Basic as an Excel function. It is listed as "radvmod" in the appendix. The first check was to compare calculations of the Excel function with calculations with the keystroke program on an HP calculator. After correcting several errors in the writeup of the equations, satisfactory agreement was obtained. The second check was to compare the Excel function, listed in the appendix as "radv", output with tables of solar insolation accepted in the field. Comparisons were made with data in tables from "Solar Heating and Cooling", Jan F. Krieder and Frank Kreith (Ref. 1). Figures 1 through 3 present comparisons on March 21 at 40 deg N Lat, for vertical, 40 deg inclination, and horizontal surfaces. Very good agreement is shown.

Figures 4 through 6 the same comparisons for June 21. The vertical surface comparison, Figure 4, is quite interesting. An E-W wall in summer does not receive direct sunlight for some time after sunrise or for some time before sunset. The tables in Ref 1 include ground reflection, so some insolation occurs before the wall gets direct sun. This is shown graphically in Figure 4, since the approximate calculation does not include ground reflection. Figures 5 and 6 show that the calculation is a little high in midday, but is still quite acceptable.

The calculations for September 21 are the same as those for March 21, so the comparisons are not repeated for September.

Figures 7 through 9 present results for December 21, the most southward travel of the sun. Calculations are just slightly below the table values for a vertical wall. Calculations for the inclined and horizontal surfaces are not quite as good, as much as

14 per cent low at noon, but are still acceptable. Fortunately, the primary interest in this application is in the vertical surface.

The second requirement of this analysis is a solution for the coupled differential equations of the heat transfer in the window and cover. Some of the solar input is reflected from the cover, some is adsorbed by the cover, another portion is absorbed by the lead came and the glass of the window, with the remainder adsorbed by the interior structure.

The first step in satisfying the second requirement is to evaluate the physical processes in as complete a manner as possible. First, some of the solar input is reflected from the cover. Reflection from a transparent medium is a function of the refractive index and the angle of incidence of the solar input. The Handbook of Chemistry and Physics (Ref. 2) gives a table on p. 2437 for reflection for a refractive index of 1.55 (appropriate for both glass and Lexan). Figure 10 shows that a simple exponential equation is a good representation of the results calculated using Fresnel's formula. Note that the solar input has a constantly changing angle with respect to the cover, so an equation is required.

Second, an estimate of the solar energy adsorbed by the Lexan is required. A phone call to GE Plastics Structured Products Technical Sales produced a fax of a test report on the exposure of UV-coated and uncoated Lexan to the sun (Ref. 3). Figure 11 presents the data from the fax and an equation which is a reasonable fit to the data. An extrapolation of the data on uncoated Lexan to 14 years (exposure time of these covers), accounting for reflection, gave a value of 0.20 for adsorption of the solar input by the cover. A rough check of transmission with a light meter was consistent with this value, setting one of the constants in the equations.

Third, an estimate of the solar energy adsorbed by the stained glass is required. First, from the average length in one square foot of the window, about 20% of the area is covered by lead, adsorbing essentially all of the solar input. A rough check of transmission through several samples of medium density stained glass gave an average of about 25% transmission. The effect of reflection and the complete adsorption of the lead are approximately in balance, so 75% was taken for the adsorption of the stained glass. Note that light meter measurements on individual windows would be desirable. Two thermal inertia constants were computed, based of the mass and specific heat of one square foot of the Lexan cover (0.55 BTU/deg F) and one square foot of the window (0.36 BTU/deg F). Other constants required were determined by the geometry of the windows (thickness of air space, window area, vent area in and out, vertical distance between the vent areas), the properties of air, and the gravitational constant. Average atmospheric pressure was given by weather reports for O'Hare Airport (Ref. 4).

With the best estimates or determination of the constants complete, the next step is to select the heat transfer equations to be applied to the components of the window and cover system. First, the heat transfer from the interior to the stained glass window is driven by natural convection. The flow velocity across the window is a function of the temperature difference between the window and the interior. Marks "Mechanical Engineers' Handbook (Ref. 5, p 374) gives an expression for natural convection for a vertical surface over one foot high (Eq. 1).

$$U_{is} = 0.27 \cdot \sqrt[4]{|T_i - T_s|} \tag{1}$$

$$\begin{aligned} U_{is} &= natural\ convection\ heat\ transfer \sim BTU / ft^2 / hr / \deg F \\ T_i &= inside\ temperature \sim \deg F \\ T_s &= stained\ glass\ window\ temperature \sim \deg F \\ |T_i - T_s| &= absolute\ value\ of\ the\ temperature\ difference \sim \deg F \end{aligned}$$

Then the heat transfer from the room to one square foot of the window is given by Equation 2.

$$Q_{is} = U_{is} \cdot (T_i - T_s) \tag{2}$$

$$Q_{is} = heat\ transfer \sim BTU / ft^2 / hr$$

Note that the equation is written in terms of heat transfer to the window. That is, if T_i the interior temperature, is hotter than T_s, the window temperature, the window is gaining heat from the interior. If the window temperature is hotter than the interior temperature, the window is losing heat to the interior.

The next step is the heat transfer from the window to the air space to the cover. Since the thermal inertia of the air in one square foot of air space is very small, the heat flow across the air space can be assumed to be quasi-steady. That is, the temperature of the air space is approximately equal to the average temperature of the window and the cover. Small transient deviations may be produced by venting. "Heat Loss Through Stained Glass Windows", by Wayne Simon (Ref. 6) presents the variation in resistance factor for enclosed air spaces for depths up to three quarters of an inch, and states that the resistance is constant for larger depths. As a convenience for this program, this data has been fitted with an exponential equation, Figure 12, so that the program constant can be just the depth of the air space. Now the heat transfer from the window to the cover is, except for the effect of venting, equal to the heat transfer from the window to the air space and to the heat transfer from the air space to the cover. From Figure 12, Equation 3, for R_{sc}, the resistance to heat flow from the stained glass window to the cover, can be written.

$$R_{sc} = 0.83 \cdot (1 - e^{6.1 \cdot D_a}) \tag{3}$$

$$D_a = air\ space\ depth \sim in$$

Then, since the U-factor is just the reciprocal of the R-factor,

$$U_{sc} = \frac{1}{R_{sc}} \tag{4}$$

And

$$Q_{sc} = (T_s - T_c) \cdot U_{sc} \tag{5}$$

$$Q_{sc} = heat\ from\ s\ to\ c \sim BTU / ft^2 / hr$$

Therefor, the heat transfer from the air space to the stained glass window can be written:

$$Q_{as} = 2 \cdot (T_a - T_s) \cdot U_{sc} \tag{6}$$

The last input to the stained glass window is the portion of the solar radiation adsorbed by the window. this is given in Equation 7.

$$RAD_s = (solar\ \ insolation) \cdot (1 - refl) \cdot (1 - cover\ \ adsorb) \cdot (window\ \ adsorb) \tag{7}$$

Then the differential equation for the temperature of the window can be written:

$$\frac{dT_s}{dt} = \frac{Q_{as} + Q_{is} + RAD_s}{TM_s} \tag{8}$$

$$TM_s = thermal\ \ mass\ \ of\ \ window \sim BTU\ /\ ft^2\ /\ \deg\ F$$

The next task is to develop the differential equation for the temperature of the air space. The first two terms are simple, since the heat transfer to the air space is just the negative of the heat transfer from the window and the cover to the air space, as shown in Equations 9 and 10.

$$Q_{sa} = -Q_{as} = 2 \cdot (T_s - T_a) \cdot U_{sc} \tag{9}$$

$$Q_{ca} = -Q_{ac} = 2 \cdot (T_c - T_a) \cdot U_{sc} \tag{10}$$

The nest step is to develop the equation for the effect of venting. When the air in the air space is hotter (or colder) than the outside air, the difference in density creates a buoyancy force which, if venting area is provided at the top and the bottom of the air space, causes a flow of air. Marks Handbook (Ref. 5, p 1120) gives an equation for the draft (pressure difference) as a function of height (vertical distance between inlet and outlet vent area), ambient pressure, outside temperature, and the air space temperature. Converting to the units of this work, it is given as Equation 11

$$D = 0.0188 \cdot P \cdot H \cdot \left(\frac{1}{T_o + 460} - \frac{1}{T_a + 460} \right) \tag{11}$$

$$D = draft \sim \frac{lb}{ft^2}$$

$$P = ambient\ \ pressu \sim \frac{lb}{ft^2}$$

$$H = vertical\ \ distance\ \ between\ \ inlet\ \ and\ \ outlet \sim ft$$

$$T_o = ambient\ \ temperature \sim \deg F$$

$$T_s = air\ \ space\ \ temperature \sim \deg F$$

Note that the quantity "460" added to the temperatures converts to an absolute temperature scale, degrees Rankine. Now the steady state flow through the air space represents a balance between the draft and the pressure loss due to flow. For the usual case, where the area of the vents is much smaller than the flow area through the air space, the loss can be evaluated as a fraction of the dynamic pressure through the inlet and outlet, as given in Equation (12).

$$D = \eta \cdot \frac{\rho}{2 \cdot G} \cdot (V_i^2 + V_o^2) \tag{12}$$

$$\eta = loss \ \ ratio$$
$$\rho = density \sim \frac{lb}{ft^3}$$
$$G = gravitational \ \ acceleration \sim \frac{ft}{\sec^2}$$
$$V_i = vent \ \ inlet \ \ velocity \sim \frac{ft}{\sec}$$
$$V_o = vent \ \ outlet \ \ velocity \sim \frac{ft}{\sec}$$

Using mass conservation to express vent velocity ratio in terms of vent area ratio, Equation 13 can be developed.

$$V_i = \frac{D}{|D|} \cdot \sqrt{\frac{|D|}{\eta \cdot \frac{\rho}{G} \cdot (1 + \frac{A_i^2}{A_o^2})}} \tag{13}$$

Note that, in order to compute correctly if the air space temperature is colder than the outside air, the sign of D has been moved outside the square root. This could occur in an air conditioned building with small solar input. The rate of heat loss (or gain) due to the vent flow can now be computed with Equation 14.

$$Q_{vent} = \frac{3600 \cdot \rho \cdot A_i \cdot V_i \cdot (T_o - T_a)}{A_{sgw}} \tag{14}$$

$$Q_{vent} = venting \ \ heat \ \ loss \sim BTU \,/\, ft^2 \,/\, hr$$
$$A_{sgw} = area \ \ of \ \ stained \ \ glass \ \ window \sim ft^2$$

Now the differential equation for the temperature of the air space can be written as Equation 15.

$$\frac{dT_a}{dt} = \frac{Q_{sa} + Q_{ca} + Q_{vent}}{TM_a} \tag{15}$$

$$TM_a = thermal\ \ mass\ \ of\ \ air \sim BTU\ /\ ft^2\ /\deg\ F$$

The last step is to develop the differential equation for the temperature of the cover. Reference 6 presented a table of R-Factor for the exterior of a building as a function of wind speed. For this program, wind speed is a variable, so an equation is required. Figure 13 presents a comparison of Equation 16 with the data from Reference 6.

$$R_{oc} = \frac{0.68}{(1 + 0.2 \cdot V)} \tag{16}$$

$$V = wind\ \ speed \sim mph$$

Again, since the U-factor is just the reciprocal of the R-factor,

$$U_{oc} = \frac{1}{R_{oc}} \tag{17}$$

And

$$Q_{oc} = (T_o - T_c) \cdot U_{oc} \tag{18}$$

$$Q_{oc} = heat\ \ from\ \ outside\ \ to\ \ cover \sim BTU\ /\ ft^2\ /\ hr$$

The heat transfer from the air space to the cover, Q_{ac}, has already been developed in Equation 10, so the remaining term, the portion of the solar radiation adsorbed by the cover, is given by Equation (19).

$$RAD_c = (solar\ \ insolation) \cdot (1 - refl) \cdot (cover\ \ adsorb) \tag{19}$$

Now the differential equation for the temperature of the cover can be written as Equation 20

$$\frac{dT_c}{dt} = \frac{Q_{ac} + Q_{oc} + RAD_c}{TM_c} \tag{20}$$

$$TM_c = thermal\ \ mass\ \ of\ \ cover \sim BTU\ /\ ft^2\ /\deg\ F$$

Note that the three differential equations, (8,15, and 20) are not independent. Each temperature appears in the differential equation of the others, making this a coupled set of differential equations. The solution is obtained by a very fine time scale simultaneous integration (12 second interval) with ten repetitions of the complete integration. This Excel function is listed in the appendix as "Tinteg".

The temperature time histories of the components then depend on the natural convection from the window to the interior of the building, the mixed conduction/convection between the window and the air space and between the air

space and the cover, the forced convection from the cover to the exterior atmosphere, and, for the case of venting, the removal of heat from the air space by buoyancy driven convection. In actuality, the temperatures also vary with vertical and horizontal location in the window and cover. In order to simplify the analysis, it is assumed the temperature at all locations in the window and cover is uniform. Another way of stating this restriction is that the analysis calculates the space averaged window, airspace, and cover temperatures.

The third requirement for the analysis is a knowledge of the local weather. First, the heat transfer from the cover to the outside atmosphere varies by a factor of four as the wind speed varies from 0 to 15 mph, a very modest wind. For this purpose, data for O'Hare airport was obtained from the National Climactic Data Center. No direct measurement of solar insolation was found, but the weather descriptions were used to adjust the variation in transmission through the atmosphere (cloud cover, fog, blowing snow, etc.).

First, the data was searched to find a day as clear as possible. January 26. 1996 had a little fog in the morning, but was otherwise clear. This made possible a test of the program with minimum ambiguity. January 27 was added as an example of a day with much smaller solar input. Figure 14 compares the measured air space temperature of two similar windows in the same wall, differing only in venting. One is sealed, the other has 0.147 square inch vents at the top and bottom of the cover. The effect of venting is not large, but the peak temperatures are significantly reduced. This probably indicates that the vent area is on the small side. Larger vent areas will be investigated with the computer model.

Figure 15 makes the same comparison for the measurements of humidity. The vented window has significantly lower humidity during solar insolation, even though the humidity levels were fairly low in both windows. The equations for humidity have not yet been included in the computer model, so the effect of larger vent areas on humidity cannot yet be computed.

Figure 16 compares measured and calculated air space temperatures for the unvented window. Figure 17 presents the adjusted air transmission factor which was used to compute the calculated temperature. No independent measure of the solar insolation was available, so the magnitude of the dips in the transmission factor have been determined by the measured air space temperature. The location in time of the dips is consistent with the weather reports. Figure 18 presents the measured wind speed at O'Hare airport, which is also a vital part of the calculation through its effect on exterior heat transfer.

Comparison of measured and calculated air space temperatures for the vented window is given in Figure 19. Note that the adjusted air transmission factor is that of Figure 17 and, of course, the measured wind speed is that of Figure 18. The agreement is not quite as good as that for the unvented window. This may indicate that significant vertical variation in the air space temperature may occur with venting.

Figure 20 presents the computed effect of increasing the vent area. It is seen that the vent area of the test was too small to provide significant reduction of temperature from that of an unvented window. .

Conclusions and Recommendations

Preliminary analysis of the data is quite encouraging. The only missing factor is an independent measurement of solar insolation. The humidity calculations can be added to the program at a later time. A real test of the humidity calculations will require data from the southeast region, where air conditioned interiors and a hot humid exterior atmosphere create real humidity problems. Finally, for a reliable prediction of the effect of protective glazing on stained glass windows, the computed results must be compared with measured data for several different installations. The installations should be chosen with consideration of the locations for which the National Climactic Data Center has detailed weather records.

References

1. Kreider, Jan F. and Kreith, Frank, "Solar Heating and Cooling", McGraw-Hill, 1975

2. Hodgman, Charles D. (editor), "Handbook of Chemistry and Physics" Thirty-Second Edition, Chemical Rubber Publishing Co., 1950

3. Personal Communication from Lillian Barker, Technical Sales, GE Plastics Structured Products, Pittsfield, Ma., May, 1996.

4. Weather Reports for O'Hare airport, November, 1995 through February, 1996, U.S. Department of Commerce, National Climactic Data Center, Asheville, Md

5. Marks, Lionel S. (editor), Mechanical Engineers' Handbook, McGraw-Hill Book Co., Fifth Edition, 1951

6. Simon, Wayne E., "Heat Loss Through Stained Glass Windows", Stained Glass, Summer, 1981

Figure 1 - Solar Insolation in BTU/sq ft/hr (March 21, 40 deg N Lat, Vertical E-W Wall)

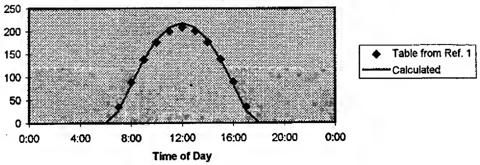

Figure 2 - Solar Insolation in BTU/sq ft/hr (March 21, 40 deg N Lat, E-W wall at 40 deg Inclination)

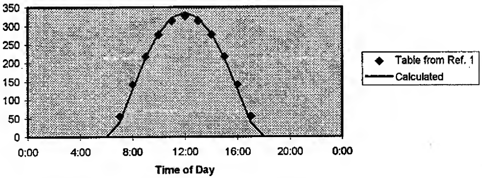

Figure 3 - Solar Insolation in BTU/sq ft/hr (March 21. 40 deg N Lat, Horizontal Surface)

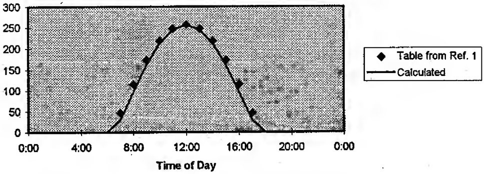

Figure 4 - Solar Insolation in BTU/sq ft/hr (June 21, 40 deg N Lat, Vertical E-W Wall)

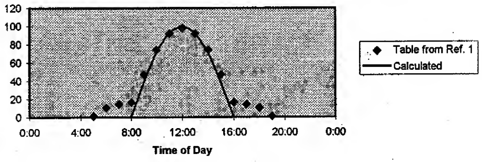

Figure 5 - Solar Insolation in BTU/sq ft/hr (June 21, 40 deg N Lat, E-W Wall at 40 deg Inclination)

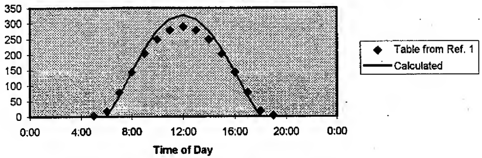

Figure 6 - Solar Insolation in BTU/sq ft/hr (June 21, 40 deg N Lat, Horizontal Surface)

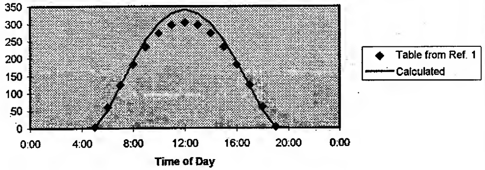

Figure 7 - Solar Insolation in BTU/sq ft/hr
(December 21, 40 deg N Lat, Vertical E-W Wall)

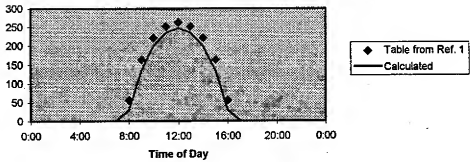

Figure 8 - Solar Insolation in BTU/sq ft/hr
(Decenber 21, 40 deg N Lat, E-W Wall at 40 deg Inclination

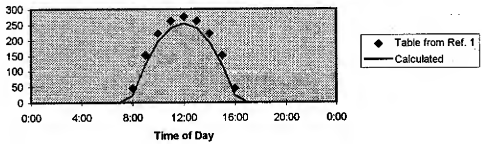

Figure 9 - Solar Insolation in BTU/sq ft/hr
(December 21, 40 deg N Lat, Horizontal Surface)

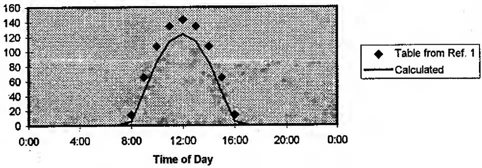

Figure 10 - Reflection from Transparent Medium With Refractive Index of 1.55 (Glass or Lexan)

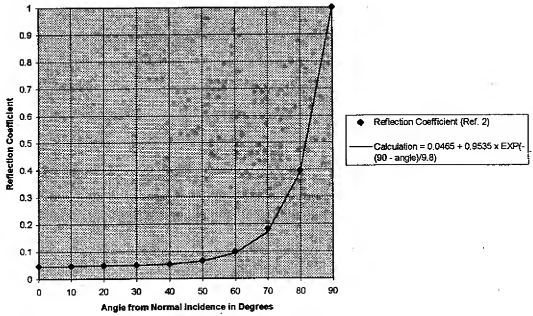

Figure 11 - Transmission of Uncoated Lexan (Data from fax from GE Plastics, Ref. 3)

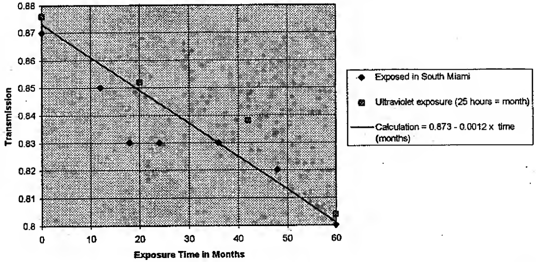

Figure 12 - R-Factor for Enclosed Air Space

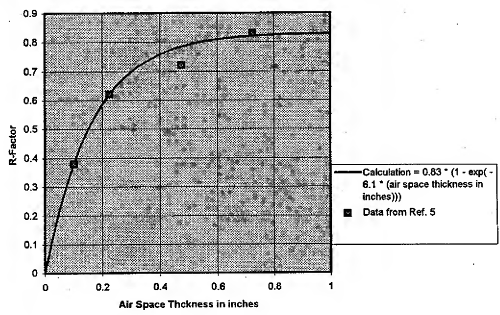

Figure 13 - R-Factor for Building Exterior

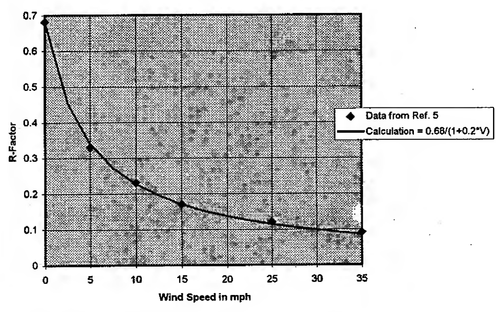

Figure 14 - Measured Air Space Temperature (unvented and vented)

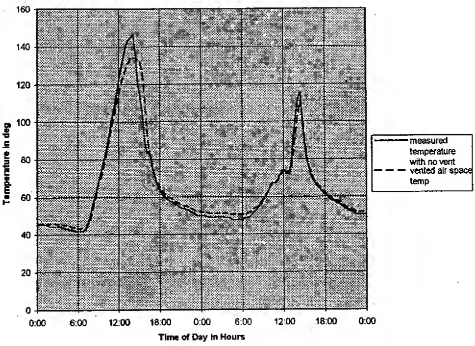

Figure 15 - Measured Air Space Humidity (unvented and vented)

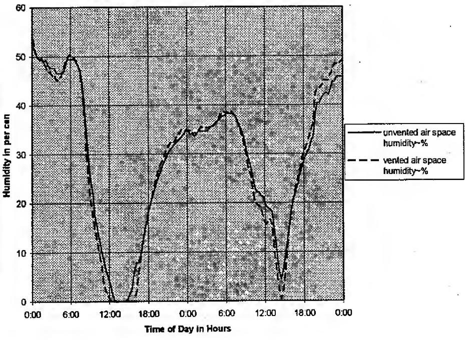

Figure 16 - Measured and Calculated Air Space Temperature (unvented)

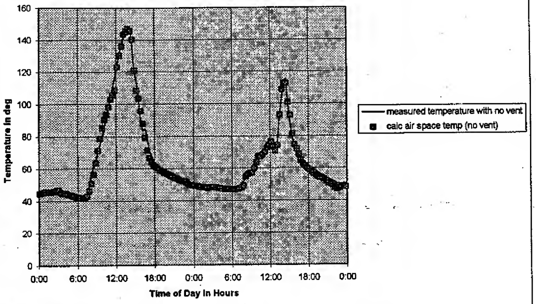

Figure 17 - Adjusted Air Transmission Factor

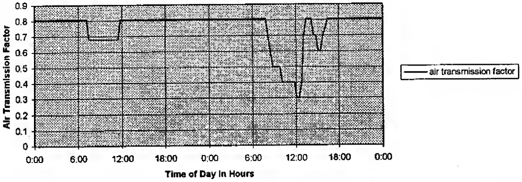

Figure 18 - Measured Wind Speed at O'Hare Airport

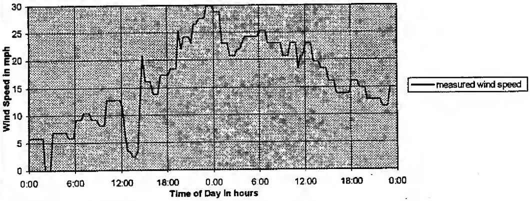

Figure 19 - Measured and Calculated Air Space Temperature (0.147 sq in vent)

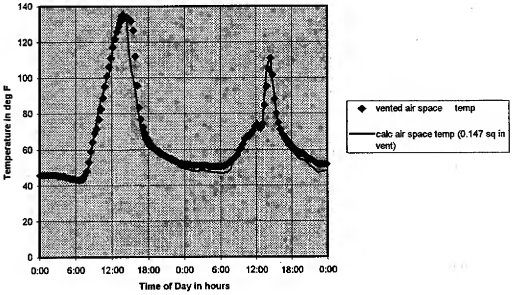

Figure 20 - Calculated Effect of Vent Ares